Nonlinear Systems and Chaos

Fall 2020

- **Instructor:** Paul Stanley, **e-mail:** *stanleyp@beloit.edu*
- **Lecture:** To be determined; expect to meet three hours per week for 15 weeks.
- **Text:** Topics in Nonlinear Dynamics for Post Novice Math or Science Students, by Paul Stanley. Draft edition; will be made available as a PDF to students at no cost. There is also a vast repository of reliable resources available on-line for reference.
- **Prerequisites:** At least a year of calculus, and a year of university level science courses, ideally with significant quantitative expectations. Some facility with either computer programming, spreadsheets, or graphing software will be useful. Coverage of the material will be at the intermediate university level and should be appropriate and accessible for junior (3rd year) math and science majors from any major.
- **Course Blurb:** Nonlinear systems and chaos is the study of systems that cannot be approximated by traditional small correction techniques. This includes population dynamics in biology, chemical equilibrium in certain systems, problems in finance, global and local weather patterns, efforts to solve planetary motion, and turbulence in fluids. This course explores the tools for addressing nonlinear systems, identifies the limits of solutions, and provides graphical and numerical techniques as a way to develop a deeper understanding of what makes something chaotic, and how to extract information from chaos.
- **Course Description:** Nonlinear Systems and Chaos is a one semester introduction to nonlinearity issues in problems in math, physics, chemistry, biology, and economics. Drawing on examples from the various subject areas, combined with with analytical, graphical, and numerical techniques, the course will explore

    1. One dimensional discrete mappings, non-linear populations dynamics;
    2. Two dimensional mappings;
    3. Area preserving systems and conservation laws and the famous KAM theorems;
    4. Dissipative systems with friction or population loss;
    5. Continuous systems and the work of Poincaré and Lorenz;
    6. Three body problems and historical concerns;
    7. Turbulence.

- **Course Goals:** By the end of the semester the student should be able

    1. to interpret and solve written problems related to the above content, discerning which presented facts and variables are relevant to the solution goal, applying the appropriate principles to the problem, and then systematically following through the process of arriving at the solution so that it is consistent with the problem statement.
    2. to evaluate solutions to nonlinear problems, either your own or presented solutions, as being dimensionally correct, numerically reasonable, and derivable from the known or stated data.
    3. to develop and execute a plan of attack to evaluate a hypothesis, for either a theoretical or experimental question, and then effectively disseminate that result to the wider community.
    4. to recognize whether a problem is best solved analytically, numerically, or graphically, and then use the techniques with reasonable estimations and approximations to find a solution.
    5. to sufficiently memorize the necessary mathematics and science tools so that the above goals can be accomplished without excessive searching for fundamental concepts.

- **Grading:**

  Attendance is required. Active participation and completion of exercises is required.

  You may work together on many of the exercises, unless explicitly told otherwise, but every student must still do their own work. For exercises where you are told to work alone, communicating with another person (another classmate, another teacher, another person on the street) will be considered academic dishonesty. The burden is on you to be aware of when you can and cannot work together; please read the disclaimers at the top of any activity very carefully.

  - Six exams (five during the semester, and one which is the final): each is worth the same amount. There will be no make up exams; the lowest exam score will be dropped, even if it is a zero. Missing an exam results in a zero. Expect exams to occur outside of normal class time; you will be on the honor system to do your own work.
  - The focus of most exams will be on the material covered since the previous exam, though it is possible that questions will address earlier material.
  - The final, however, will be cumulative.
  - The sum of all in class quizzes and activities and homework will have a total value equivalent to two exams.
  - A major design project/presentation on an aspect of nonlinear systems will have a value equivalent to one exam.

  Grading Rubric

Numeric Percentage	Description
less than 40	Work shows little knowledge of math and science relevant to the problem.
60	There is clear evidence in the work that a reasonable approach to the problem was attempted, but there are significant errors in units, formulae, or fundamental concepts. Diagrams were applicable, but sloppy, unclear, or mislabeled.
70	Either a reasonable, but incorrect, approach was attempted with few other errors; or a correct approach was presented with errors in units or math. Diagrams were adequate and may have a few errors.
80	A correct approach was presented with few errors. Diagrams were clear and correctly labeled.
90	A correct approach was presented with no more than one minor error. Words were used at times to clearly indicate how and why the derivation was performed. Diagrams were neat, accurate, and cleanly illustrated the concepts.

  Grading Scale: The minimum cutoff percentages for the final grade

A	≥ 90	B	≥ 75	C	≥ 60
A-	≥ 85	B-	≥ 70	C-	≥ 55
B+	≥ 80	C+	≥ 65	D	≥ 40

- **Accessibility:** If you have a concern about course accessibility and would like to discuss the possibility of accommodations, contact ACM at ocs@acm.edu or 312-263-5000. If you need accommodations in my class, you must bring me an Accommodation Verification Letter from the ACM office and then we will discuss specifically how to meet your needs. Please contact that office promptly, accommodations are not retroactive. Let me know if there is a problem contacting any office; I will work with you to try and resolve any issues.

  You should expect to spend six to ten hours each week outside of class reading, studying, doing homework problems, or reviewing in class examples. Staring at your notes is *not* the way to learn the material; you must attempt, and eventually succeed in, solving problems. If you don't understand, seek assistance. It is my intent to get every person to a place where they have accomplished their personal goals for the course. We can be fairly creative in how we do this, but it needs to be done proactively, not reactively.

Approximate Weekly Schedule

1. Linear systems, one dimensional mappings, simple population dynamics with rabbits, period doubling, numerical and graphical techniques.
2. The appearance of chaos, importance of period three, the tent map, fractal structures, linearization, Lyapunov exponents.
3. Two dimensional mappings, complex numbers, Mandelbrot, Julia sets, mixing
4. Fractals, perfect and imperfect, invariant sets, ergodicity, entropy
5. Area preserving mappings, Poincaré, homoclinic tangles, the Standard Map
6. Continuous systems, Lorenz, weather, chemical chaos
7. Conserved systems
8. Invariant tori, the three body problem, rings of Saturn
9. Dissipative systems, strange attractors
10. KAM Theorem (this will be the hardest math, but the most fun conceptually!)
11. Chaos in electro-mechanical systems: oscillators, circuits, inverted pendula
12. Chaos in optical systems
13. Special topics decided on by class
14. Turbulence, power laws, scaling
15. Final exam, remaining project work due